SOME INFORMATIONAL ASPECTS OF DECISION-MAKING IN MONOPOLISTIC PRODUCTION SYSTEMS

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Abstract. This paper addresses a decision problem of a monopolist producer that aims to maximize profit and operates under three distinct information conditions: certainty, risk or uncertainty. The mathematical model is formulated as the maximization of a profit function $R^{\circ}(x, u, y)$ that depends on two sets of controllable factors - the resource acquisition vector x and the supply volume vector u - and the uncontrollable demand vector y. The model constraints express the technological interdependence between consumed and purchased resources, along with upper and lower bounds for all decision variables. The analysis is structured around three hypotheses determined by the demand-supply relationship for each good j. According to the first hypothesis, where demand exceeds supply, the optimal decision is to offer the maximum available quantity. The second hypothesis where supply exceeds demand: a particular case with a linearly decreasing price function is analyzed and a double inequality is obtained that must be satisfied by the coefficient k_j that expresses the ratio between the maximum and minimum price, ensuring that the optimal solution remains within the admissible range. The third hypothesis addresses the intermediate case in which demand falls between the limits of supply; here, the generalized gradient of the profit function is used, providing the calculation basis for numerical optimization algorithms. For each situation, numerical algorithms are developed to determine the optimal supply variants. Under risk conditions, the mathematical expectation of profit is analyzed. For uncertain conditions, where a finite set of states of nature is operated, the Savage criterion for minimizing regrets is applied and a parallel iterative algorithm is described that executes $L + 1$ simultaneous processes per iteration, with guaranteed convergence under certain conditions.

Keywords: monopolist producer, stochastic demand, optimal decisions, generalized gradient, regret function

JEL Classification: C02, C61, D42, D81.

INTRODUCTION

Monopoly represents the economic agent (called monopolist producer) that operates in the space of large-scale business and within which goods are produced to satisfy the most stringent and widespread needs [1]. Namely, monopoly is that economic system that offers standardized consumer products. In economic theory and practice, it is known that the producer-monopolist can reduce product prices with the increase in quantities offered for sale. Of course, through his activity he tends to achieve certain objectives of his own. One of these objectives, but also the most frequently encountered, consists in determining such policies (decisions), which would ensure obtaining

maximum profit. It is precisely in this direction that the present paper attempts to investigate the behavior of the monopolistic producer.

MODEL FORMULATION AND DESCRIPTION

The profit function is required to be maximized.

$$R°(x,u,y) = \sum_{j=1}^{n}\left[c_j(u_j)\cdot min\{u_j,y_j\} - p_j(u_j)max\{0,u_j-y_j\}\right] - \sum_{i=1}^{m} r_i(\cdot)\ x_i \quad (1)$$

dependent on controllable factors $x=(x_1,\dots,x_i,\dots,x_m)$, $u=(u_1,\dots,u_j,\dots,u_n)$ and those of an uncontrollable nature $y=(y_1,\dots,y_j,\dots,y_n)$.

Model restrictions, in relation to control factors x and u, have the appearance:

$$\sum_{j=1}^{n} a_{ij}.u_j = x_i,\ \ i=\overline{1,m} \quad (2)$$

$$x\in X=\{x\in E^m: 0\le \underline{x_i}\le x_i\le \overline{x_i}, i=\overline{1,m}\} \quad (3)$$

$$u\in U=\{u\in E^n: 0\le \underline{u_j}\le u_j\le \overline{u_j}, j=\overline{1,n}\} \quad (4)$$

Meaning of the notation:

u_j – volume of the product type j, $(j=\overline{1,n})$– quantity supplied of the good j;

x_i – the quantity of the production factor i, $(i=\overline{1,m})$– the demand for resources of the type i;

y_j – the volume of demand for the product j, $(j=\overline{1,n})$;

$c_j(u_j)$ – the unit price of the good j, $(j=\overline{1,n})$ – depends on the quantity supplied u_j;

$p_j(u_j)$ – unit losses (expressed in monetary units), conditioned by the "overproduction" phenomenon (when $(u_j>y_j)$;

$r_i(x_i)$ - the unit price of the resource i – may depend on the quantity of the resource procured in volumex_i units;

a_{ij} – technological coefficient – expresses the quantity of resource $i$used to produce a unit of good j – constant size.

Constraints (2) express the balance between the resources used in the production process and those procured. Therefore, instead of the function $R°(x,u,y)$ in (1), the function can be considered:

$$R(u,y)=\sum_{i=1}^{n} R_j(u_j,y_j), \quad (5)$$

where $R_j(u_j,y_j)$ the expression is denoted by:

$$R_j(u_j,y_j)=c_j(u_j)\cdot min\{u_j,y_j\} - p_j(u_j)max\{0,u_j-y_j\} - \left(\sum_{i=1}^{m} a_{ij}\cdot r_i(\cdot)\right)u_j, j=\overline{1,n} \quad (6)$$

Obviously, the function $R_j(u_j,y_j)$ is presented in two analytical aspects, depending on the relationship between $u_j$and y_j (i.e. between supply and demand for the good j):

$$R_j(u_j, y_j) = \begin{cases} \left(c_j(u_j) - \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)\right) u_j, & if\ u_j \le y_j \\ \left(c_j(u_j) + p_j(u_j)\right) y_j - \left(p_j(u_j) + \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)\right) u_j, if\ u_j \ge y_j \end{cases} \quad (7)$$

Next, we analyze three hypotheses concerning different interdependencies between demand and supply.

Hypothesis I. Let $y_j \le \underline{u}_j$.

In this case the value $\frac{max}{uj} R_j(u_j, y_j)$ is reached for $u_j^* = \underline{u}_j$ and $\frac{max}{u} R(u, y) = R(\underline{u}, y)$, that is, it is rational for the monopolistic decision-maker to focus on the optimal offer option$u^* = \underline{u}$ and the optimal decision regarding the procurement of resources in terms of $x_i^* = \sum_{j=1}^{n} a_{ij} \cdot \underline{u}_j$.

*Hypothesis II*_. Let $y_j \ge \bar{u}_j$. From (7), it obviously follows that:

$$R_j(u_j, y_j) = (c_j(u_j) - \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)) \cdot u_j$$

The first order derivative of this function is:

$$\frac{dR_j(u_j, y_j)}{du_j} = \left(c_j(u_j) - \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)\right) + \left(\frac{dc_j(u_j)}{du_j} - \sum_{i=1}^{m} a_{ij} \frac{\partial r_i(\cdot)}{\partial u_j}\right) \cdot u_j$$

The expression $\sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)$ describes the cost of all the resources needed to obtain one unit of product j. To obtain real profit in relation to the good j, obviously, it is necessary that the difference between price and cost be strictly positive:

$$c_j(u_j) - \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot) > 0 \quad (8)$$

If the equation $\frac{dR_j(u_j, y_j)}{du_j} = 0$ has the size as its solution $\tilde{u}_j$ then it obviously satisfies the equality:

$$\tilde{u}_j = \frac{\left(c_j(\tilde{u}_j) - \sum_{i=1}^{m} a_{ij} \cdot r_i(\cdot)\right)}{\left(\sum_{i=1}^{m} a_{ij} \frac{\partial r_i(\cdot)}{\partial u_j} - \frac{dc_j(\tilde{u}_j)}{du_j}\right)}.$$

Because the size $\tilde{u}_j > 0$ requires the expression $\sum_{i=1}^{m} a_{ij} \frac{\partial r_i(\cdot)}{\partial u_j} - \frac{dc_j(\tilde{u}_j)}{du_j}$ to be greater than zero.

In particular, if resource prices are fixed, it follows that $\frac{dc_j(u_j)}{du_j} < 0$ and therefore, as supply increases, the price of the good must decrease.

Second order derivative:

$$\frac{d^2 R_j(u_j, y_j)}{du_j^2} = 2\left(\frac{dc_j(u_j)}{du_j} - \sum_{i=1}^{m} a_{ij} \frac{\partial r_i(\cdot)}{\partial u_j}\right) + \left(\frac{d^2 c_j(u_j)}{du_j^2} - \sum_{i=1}^{m} a_{ij} \frac{\partial^2 r_i(\cdot)}{\partial u_j^2}\right) \cdot u_j \quad (9)$$

Remark 1. Analyzing (9), we can conclude: if the price function $c_j(u_j)$ is concave and decreasing on the interval $[\underline{u}_j, \overline{u}_j]$, and the function $r_i(\cdot)$ is convex and increasing (or is a constant quantity), then the profit function with respect to the good j, $R_j(u_j, y_j)$, is concave on $[\underline{u}_j, \overline{u}_j]$, (figure 1). Under these conditions, the respective second-order derivative is negative on $[\underline{u}_j, \overline{u}_j]$.

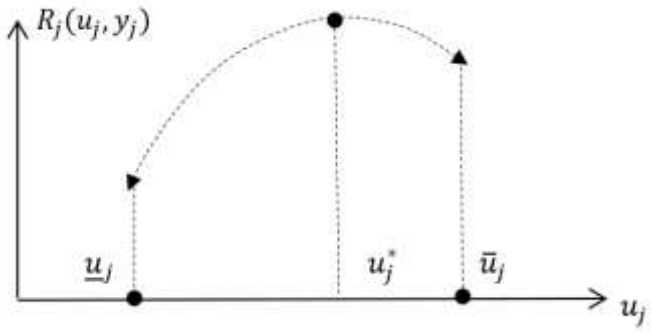


Figure 1. Concave representation of the function $R_j(u_j, y_j)$;u_j^* - the maximum point.

Particular case. Let the price $c_j(u_j)$ be a decreasing linear function with respect to $u_j$on the interval $[\underline{u}_j, \overline{u}_j]$, (figure 2).

$c_j(u_j) = \overline{c}_j - (\overline{c}_j - \underline{c}_j) \cdot (u_j - \underline{u}_j)/(\overline{u}_j - \underline{u}_j)$

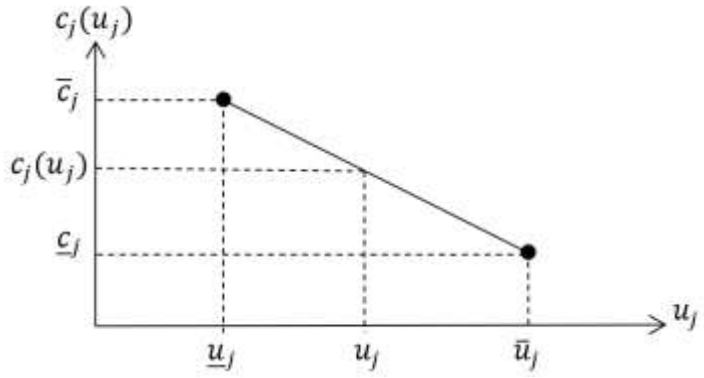


Figure 2. Graphical representation of price $c_j(u_j)$ as a linear function over the interval $[\underline{u}_j, \overline{u}_j]$

Also, it will be assumed that price $r_i(\cdot) = r_i = constant$ and $y_j \geq \overline{u}_j$.

Thus, the function $R_j(u_j, y_j) = \left(c_j(u_j) - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) \cdot u_j$ takes on the following appearance:

$$R_j(u_j, y_j) = \left(\frac{\overline{c}_j \overline{u}_j - \underline{c}_j \underline{u}_j}{\overline{u}_j - \underline{u}_j} - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) \cdot u_j - \frac{\overline{c}_j - \underline{c}_j}{\overline{u}_j - \underline{u}_j} \cdot u_j^2$$

Let us denote $A_j = \left(\frac{\overline{c}_j \overline{u}_j - \underline{c}_j \underline{u}_j}{\overline{u}_j - \underline{u}_j} - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right)$, $B_j = \frac{\overline{c}_j - \underline{c}_j}{\overline{u}_j - \underline{u}_j}$

The derivative of the function $R_j(u_j, y_j)$ with respect to u_j is $A_j - 2B_j \cdot u_j$.

$$\frac{dR_j(u_j, y_j)}{du_j} = \left(\frac{\overline{c}_j \overline{u}_j - \underline{c}_j \underline{u}_j}{\overline{u}_j - \underline{u}_j} - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) - 2\frac{\overline{c}_j - \underline{c}_j}{\overline{u}_j - \underline{u}_j} \cdot u_j$$

Considering the equation $\frac{dR_j(u_j,y_j)}{du_j} = 0$, its solution (denoted by u_j^*) is determined elementary:

$$u_j^* = A_j/2B_j.$$

Assuming the values $\overline{u}_j$ and are known $\underline{u}_j$, we will study the correlation between the quantities $\overline{c}_j$ and $\underline{c}_j$ so that the quantity u_j^* falls within the interval [$\overline{u}_j$, $\underline{u}_j$]. Obviously,

$$u_j^* = \left[\left(\overline{c}_j\overline{u}_j - \underline{c}_j\underline{u}_j\right) - \left(\overline{u}_j - \underline{u}_j\right)\sum_{i=1}^{m} a_{ij}r_i\right]/2\left(\overline{c}_j - \underline{c}_j\right) \quad (10)$$

So, according to what was mentioned, u_j^* it must respect the inequality:

$$\underline{u}_j \le u_j^* \le \overline{u}_j \quad (11)$$

From (10) and (11) the inequality follows:

$$2\left(\overline{c}_j - \underline{c}_j\right)\underline{u}_j + \left(\overline{u}_j - \underline{u}_j\right)\sum_{i=1}^{m} a_{ij}r_i \le \overline{c}_j\overline{u}_j - \underline{c}_j\underline{u}_j \le 2\left(\overline{c}_j - \underline{c}_j\right)\overline{u}_j + \left(\overline{u}_j - \underline{u}_j\right)\sum_{i=1}^{m} a_{ij}r_i \quad (12)$$

Without loss of generality, it can be considered that $\overline{c}_j = k_j \cdot \underline{c}_j$ for some value of $k_j \ge 1$. Thus, from (12) it can be deduced that:

$$\frac{\left(\overline{u}_j - \underline{u}_j\right)\sum_{i=1}^{m} a_{ij}r_i}{k_j\overline{u}_j - \left(2k_j - 1\right)\underline{u}_j} \le \underline{c}_j \le \frac{\left(\overline{u}_j - \underline{u}_j\right)\sum_{i=1}^{m} a_{ij}r_i}{\left(2 - k_j\right)\overline{u}_j - \underline{u}_j} \quad (13)$$

Inequality (13) can occur if the following relationship is satisfied:

$$k_j\overline{u}_j - \left(2k_j - 1\right)\underline{u}_j \ge \left(2 - k_j\right)\overline{u}_j - \underline{u}_j > 0 \quad (14)$$

To ensure (8) it is also necessary that:

$$\frac{\overline{u}_j - \underline{u}_j}{k_j\overline{u}_j - \left(2k_j - 1\right)\underline{u}_j} \ge 1 \text{ and } \frac{\overline{u}_j - \underline{u}_j}{\left(2 - k_j\right)\overline{u}_j - \underline{u}_j} \ge 1, j = \overline{1,n}, \quad (15)$$

From (14) follows the inequality $k_j(\overline{u}_j - \underline{u}_j) \ge \overline{u}_j - \underline{u}_j$ or $k_j \ge 1$.

From the fact that $\left(2 - k_j\right)\overline{u}_j - \underline{u}_j > 0$ we obtain $k_j < 2 - \underline{u}_j/\overline{u}_j$. So (14) leads to the double inequality:

$$1 < k_j < 2 - \underline{u}_j/\overline{u}_j \quad (16)$$

From the first inequality (15) it follows that the coefficient k_j again must satisfy relation (16), except when $\overline{u}_j < 2\underline{u}_j$. So, if $\overline{u}_j < 2\underline{u}_j$ it is obtained that: $1 < k_j < \underline{u}_j/(2\underline{u}_j - \overline{u}_j)$.

The second inequality in (15) implies inequality (16). Summarizing what has been obtained, it is found that k_j must respect the inequality:

$$1 < k_j < \min\left\{\frac{\underline{u}_j}{2\underline{u}_j - \overline{u}_j}; 2 - \frac{\underline{u}_j}{\overline{u}_j}\right\},$$

if $\overline{u}_j < 2\underline{u}_j$. It is easy to deduce that also for the case $\overline{u}_j < 2\underline{u}_j$, k_j will comply with (16). In other words, even in the situation $\overline{u}_j < 2\underline{u}_j$, $\min\left\{\frac{\underline{u}_j}{2\underline{u}_j - \overline{u}_j}; 2 - \frac{\underline{u}_j}{\overline{u}_j}\right\} = 2 - \frac{\underline{u}_j}{\overline{u}_j}$.

Indeed, considering the difference (and noting with $\alpha = \underline{u}_j/\overline{u}_j$) $\underline{u}_j/(2\underline{u}_j - \overline{u}_j) - (2 - \underline{u}_j/\overline{u}_j) = 1/(2 - 1/\alpha) - (2 - \alpha) = 2(\alpha - 1)^2/(\alpha\left(2 - \frac{1}{\alpha}\right)) > 0$ because $\alpha \in \left(\frac{1}{2}; 1\right)$.

From this it follows that $2 - \underline{u}_j/\overline{u}_j < \underline{u}_j/(2\underline{u}_j - \overline{u}_j)$ for $\overline{u}_j < 2\underline{u}_j$.

Thus, regardless of whether the relationship $\overline{u}_j < 2\underline{u}_j$ or $\overline{u}_j \geq 2\underline{u}_j$, the coefficient k_j must respect inequality (16).

Remark 2. If the decision maker applies a policy of decreasing the price linearly over the interval $[\underline{u}_j, \overline{u}_j]$ and the maximum price (which corresponds to the supply volume $\underline{u}_j$) is $\bar{c}_j$, then, for the maximum supply of $\overline{u}_j$units, he can set the minimum price $\underline{c}_j = \bar{c}_j / k_j$, where k_j he cannot exceed the quantity of $(2 - \underline{u}_j/\overline{u}_j)$ units. Otherwise, he would not obtain the expected result.

Therefore, according to the analytical study, condition (16) determines the range (13) for setting the minimum price $\underline{c}_j$, to ensure that the optimal offer u_j^* falls within the a priori indicated range $[\underline{u}_j, \overline{u}_j]$.

Remark 3. If the demand for the good $j$were known a priori y_j, then one could choose the prices $\underline{c}_j$ and $\overline{c}_j$ (i.e. the coefficient k_j) so as to analytically determine the optimal supply u_j^* and the respective demand for the resource $x_i^* = \sum_{j=1}^{n} a_{ij} \cdot u_j^*$.

Hypothesis III. $\underline{u}_j \leq y_j \leq \overline{u}_j$ From the overview of the function:

$$R_j(u_j, y_j) = c_j(u_j) \min\{u_j, y_j\} - p_j(u_j) \max\{0, u_j - y_j\} - \left(\sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) u_j,$$

it results that for the case $y_j \in [\underline{u}_j, \overline{u}_j]$ the following concrete presentation is obtained:

$$R_j(u_j, y_j) = \begin{cases} \left(c_j(u_j) - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) u_j, & if\ u_j \leq y_j \\ \left(c_j(u_j) + p_j(u_j)\right) y_j - \left(p_j(u_j) + \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) u_j, & if\ u_j > y_j. \end{cases}$$

Thus, the generalized derivative [2]:

$$dR_j(u_j, y_j) = \begin{cases} \left(c_j(u_j) - \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) + \left(\frac{dc_j(u_j)}{du_j} - \sum_{i=1}^{m} a_{ij} \frac{dr_i(\cdot)}{du_j}\right) \cdot u_j, & if\ u_j \leq y_j \\ \left(\frac{dc_j(u_j)}{du_j} + \frac{dp_j(u_j)}{du_j}\right) \cdot y_j - \left(p_j(u_j) + \sum_{i=1}^{m} a_{ij} r_i(\cdot)\right) - \left(\frac{dp_j(u_j)}{du_j} + \sum_{i=1}^{m} a_{ij} \frac{dr_i(\cdot)}{du_j}\right) \cdot u_j, & if\ u_j \geq y_j \end{cases} \quad (17)$$

The above expressions (17) represent the calculation formulas of the generalized gradient components [2], necessary for the subsequent implementation of numerical algorithms for solving the monopolist problem under conditions of certainty and under conditions of risk.

In the deterministic situation (the demand is known) the algorithm looks like this:

$$u^{k+1} = P_U\left(u^k - h_k g(u^k, y)\right), k = 0,1, \ldots,$$

where the *jth component* of the vector $g(u^k, y)$is calculated according to the rule:

$$g_j(u_j^k, y_j) = \begin{cases} \dfrac{dR_j(u_j^k, y_j)}{du_j}, & if\ \varphi(u^k) \leq \delta_k \\ \dfrac{d\varphi(u^k)}{du_j}, & if\ \varphi(u^k) > \delta_k \end{cases}$$

The functions $\varphi(u^k) = \max\{\varphi_i^1(u), \varphi_i^2(u)\}$, where:

$$\varphi_i^1(u) = \sum_{i=1}^{n} a_{ij} u_j - \overline{x_i}, \qquad \varphi_i^2(u) = \underline{x_i} - \sum_{i=1}^{n} a_{ij} u_j$$

Remark 4. Tolerance threshold [4] δ_k can be taken equal to zero, because, as follows from the definition of the functions φ_i^1 and φ_i^2, the function $\varphi(u)$ satisfies the Slater condition.

In the risk situation, where the information regarding the demand y has a random nature, the distribution law $Q_y(t) = Prob\{y_1 < t, \dots, y_j < t_j, \dots, y_n < t_n\}$ on a certain area $Y = \{y = (y_1, y_j, y_n): \underline{y_j} \le y_j \le \overline{y_j}\}$, and the goal of the monopolist producer consists in determining such a level of supply u^*, for which the average total profit would reach the maximum value:

$$M_y[R(u^*, y)] = max M_y[R(u, y)] ,$$

where is $M_y[R(u, y)]$ the mathematical expectation of profit for the offer variant represented by the vector $u = (u_1, \dots, u_j, \dots, u_n)$, the algorithm represents a modification of the direct stochastic programming method (stochastic gradient projection method). Namely:

$$u^{k+1} = P_U\left(u^k + h_k g(u^k, y^k)\right), k = 0,1,2, \dots, \dots$$

The sequence $y^0, y^1, \dots, y^k, \dots$ is made up of independent simulations of the random vector y obtained according to the probability distribution $Q_y(t)$. The numerical sequences h_k and δ_k, both in the deterministic and risk situations, must meet the following requirements:

$$h_k > 0, h_k \to 0, \delta_k \ge 0, \delta_k \to 0,$$

$$\sum_{k=0}^{\infty} h_k \delta_k = \infty, \frac{h_k}{\delta_k} \to 0$$

In the condition, where uncertainty prevails, regarding the behavior of demand, it will be assumed that the vector y, which represents demand, is associated with a finite number of states $y^1, y^2, \dots, y^l, \dots, y^L$ of nature. These being de facto all possible variants of manifestation of demand. Likewise, it will be admitted that the monopolist as a decision maker, tries to avoid considerable deviations between the expected and the real result. Therefore, it is about the regret function, defined in terms of profit. For the demand variant $y^l = (y_1^l, \dots, y_j^l, \dots, y_n^l)$ the function $R(u, y^l) = \sum_{j=1}^n R_j(u_j, y_j^l)$, and

$$R_j(u_j, y_j^l) = c_j(u_j) \cdot min\{u_j, y^l\} - p_j(u_j) \cdot max\{0; u_j - y_j^l\} - \left(\sum_{i=1}^m a_{ij} \cdot r_i(\cdot)\right) u_j, l = \overline{1, L}.$$

For each $l = \overline{1, L}$ will be denoted by: $R^l = \max\limits_{u \in U} R(u, y^l)$, $\overline{R}(u, y^l) = (R^l - R(u, y^l))$ - the regret function that corresponds to the demand y^l, $R_s(u) = \max\limits_{1 \le l \le L} \overline{R}(u, y^l)$ – called Savage's regret function. So, $R_s(u)$ it is the maximum deviation between the expected and the real profit, if the supply were to occur u and the demand were to be realized in the most unfavorable variant. In the vision of the criterion of minimizing regrets, the monopolist is to determine such a variant of supply $u^* \in U$, for which the Savage function would be realized with the minimum possible variant:

$$R_s(u^*) = \min_{u \in U} R_s(u).$$

Description of the algorithm for determining the optimal offer variant u^* and the minimal regret value $R_s(u^*)$.

The algorithm consists of carrying out, at each iteration, $L+1$ processes, in parallel, of the form:

$$\begin{cases} u_{(l)}^{k+1} = \Pi_U\left(u_{(l)}^k + h_{(l)k} \cdot \eta_{(l)}^k\right), l = \overline{1,L} \\ u^{k+1} = \Pi_U(u^k - h_k \cdot \eta^k), \end{cases} \tag{18}$$

$$\text{where: } \eta_{(l)}^k = \begin{cases} R_u'\left(u_{(l)}^k, y^l\right) / \left\| R_u'\left(u_{(l)}^k, y^l\right) \right\|, \text{for } R_u'\left(u_{(l)}^k, y^l\right) \neq 0 \\ 0, \text{ otherwise.} \end{cases} \tag{19}$$

$$\eta^k = \begin{cases} \left(R_S^k(u^k)\right)_u' / \left\| \left(R_S^k(u^k)\right)_u' \right\|, \text{ for } \left(R_S^k(u^k)\right)_u' \neq 0 \\ 0, \quad \text{otherwise.} \end{cases} \tag{20}$$

Here $R_u'\left(u_{(l)}^k, y^l\right)$ represents some generalized gradient of the function $R(u, y^l)$ for $u = u_{(l)}^k$, and $\left(R_S^k(u^k)\right)_u'$ - the generalized gradient of the function:

$$R_S^k(u) = \max_{1 \le l \le L} \left\{ R\left(u_{(l)}^k, y^l\right) - R(u, y^l) \right\}$$

calculated for the case $u = u^k$.

What about the strings $h_{(l)k}$, $l = \overline{1,L}$ and h_k, it is necessary to satisfy the requirements:

$$h_{(l)k}, h_k \ge 0 \,;\; h_{(l)k}, h_k \to 0 \,;\; \sum_{k=0}^{\infty} h_{(l)k}, \sum_{k=0}^{\infty} h_k = \infty. \tag{21}$$

If the domain U * - is the set of minimum points of the function $R_s(u)$, and all the conditions indicated above hold, then it can be stated [4] that:

$$\lim_{k \to \infty} \min_{u^* \in U^*} \|u^k - u^*\| = 0 \text{ and } \lim_{k \to \infty} R_S^k(u^k) = R_s(u^*) \tag{22}$$

Conditions (18)-(21) ensure the convergence of the iterative process in the following sense. With the increase of k, performing the iterations after $l = \overline{1,L}$, estimates are determined that are increasingly closer to the values of R^l. These estimates help to ensure convergences of the form (22). So, if U^* consists of several variants, then convergence towards one of the optimal variables is ensured. At the same time, the estimates $R_S^k(u^k)$ approach, with the increase of k, the minimum value of the regret function.

CONCLUSIONS

The results obtained would be a justified scientific support for the rigorous mathematical substantiation of the monopolist's decision and provide operational analytical and algorithmic tools for optimizing the offer across the entire spectrum of information environments encountered in monopolistic markets.

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