DOI: https://doi.org/10.53486/sstc2025.v2.52
CZU: 330.4:338.3

PRODUCTION OPTIMIZATION UNDER LIMITED RESOURCES: APPLICATION OF CONDITIONAL EXTREMA IN ECONOMICS

ОПТИМИЗАЦИЯ ПРОИЗВОДСТВА ПРИ ОГРАНИЧЕННЫХ РЕСУРСАХ: ПРИМЕНЕНИЕ УСЛОВНЫХ ЭКСТРЕМУМОВ В ЭКОНОМИКЕ

STERBET Anastasia, studentă, specialitatea: Contabilitate
Academia de Studii Economice din Moldova, Republica Moldova,
Chișinău, str. Bănulescu-Bodoni 61, www.ase.md
e-mail autor: sterbet.anastasia@ase.md

Аннотация. *В современном мире, когда на рынке присутствует высокая конкуренция среди фирм, а такие ресурсы, как труд, капитал и земля – ограничены, оптимизация производства является краеугольным камнем современной экономики. Данная статья нацелена на тщательное исследование методов оптимизации производства, в частности, применения метода множителей Лагранжа для улучшения рентабельности предприятия и снижения производственных издержек. Методология включала в себя анализ соответствующих работ как отечественных, так и зарубежных авторов. В ходе исследования была разработана математическая модель для конкретного предприятия хлебобулочных изделий, которое производит продукты по двум разным технологиям в условиях ограниченности ресурсов. При помощи метода множителей Лагранжа удалось посчитать оптимальный объем выпуска продукции, минимизировав затраты на производство. Несмотря на то, что данный метод успешно применяется на предприятиях для оптимизации всех сфер производства, существует ряд ограничений, с которыми сталкиваются управляющие. Например, отсутствие высококвалифицированных специалистов, обладающих достаточным количеством знаний в областях, связанных с экономической математикой, микроэкономикой и финансами, а также постоянные изменения в экономической ситуации могут затруднять использование условных экстремумов.*

Ключевые слова*: экономическая математика, оптимизация производства, метод множителей Лагранжа, ограниченность ресурсов, эффективное распределение ресурсов*

JEL CLASSIFICATION: C02, C30, C61

INTRODUCTION

In today's world where there is high competition among a great variety of companies on the market, resources are limited and tend to decrease from year to year, production optimization is the milestone of modern-day economy. It represents a complex system, which functioning fully depends on a great variety of interconnected factors. The accurate analysis, financial forecasting and solution optimization are required for effective managing of economic processes, that is why mathematical modeling becomes an integral part of modern economic research.

Production optimization plays a vital role in general Management of any plant. By improving efficiency, businesses can reduce their operational costs, which directly impacts their profitability. For example, optimizing order sequences can lead to shorter operating times and reduced total processing times, which are critical for maintaining competitiveness in the market (Joppen et.al., 2019).

Economic Mathematics gives the ability to formalize complicated phenomena, reveal linked patterns in the provided data and develop effective strategies for decision-making. With the help of methods of mathematical analysis, statistics, optimization planning and mathematical modeling it is possible to assess the impact of various factors on economic processes, minimize the risks and increase the effectiveness of economic activity of the enterprise.

Production optimization under limited resources has become one of the key directions of the application of mathematical modeling in Economics. In conditions of capital investment, land and labor deficit, plants are forced to seek for the ways of efficient allocation of available resources.

The application of different mathematical models in economic analysis not only increases the accuracy of the forecasts, but also contributes to the development of new approaches in management. This paper aims to critically explore the role of a mathematical model in finding optimal solutions using Lagrange method that offers the possibility to estimate optimal production plan if resources are limited. Methodology, utilized while conducting this research, comprised a number of theoretical and practical aspects and is guided by corresponding elaborations in this realm, conducted either in Moldova or abroad.

MAIN CONTENT

1.Analysis of materials and sources of information

Joseph Louis Lagrange (1736-1813) was one of the most prominent French mathematicians in the XVIIIth century and was interested in fundamental issues in analyses and mechanics. The development of an effective instrument that would give the possibility to resolve complex problems war required so Lagrange created his own method for a search of conditional extrema. It was so versatile that quickly became applicable in problems, where minimal or maximal value of an objective function is needed, taking into consideration a set of limits. Despite the fact that problems of optimization could be faces in a host of scientific areas, like Physics, Geometry and Mechanics, Lagrange method received recognition in Economics much later. Economic experts started to implement it for modeling of economic problems connected with production optimization under limited resources, reaching high productivity and a decrease in expenses only in late XIX – early XXth centuries (Miniuc, 2002).

Leonid Kantorovich (1912-1986) was one of the most prominent Soviet experts in Economics and was the pioneer in elaborating linear problems with the usage of Lagrange method for optimization processes. In 1975, he received Nobel Prize in Economics for contribution in optimal resource allocation theory. In paper "The Best Use of Economic Resources" (1960), Kantorovich applied Lagrange method for resolving linear problems in Economics. In particular, this method was mentioned in a section, dedicated to maximal problem execution under limited resources. He used conditional extrema for assessment of restrictions and their impact on optimal solution of linear problems in Economics, that allow effectively distribute resources and improve production processes (Kantorovich, 1960).

Moreover, Lagrange method can be used in not only production optimization and cost minimization, but also in microeconomics, while resolving problem of maximizing a utility function. In paper "Fundamental Methods of Mathematical Economics" (1967) American mathematic economist Alpha C. Chiang used conditional extrema in selling good in the market. He showed, that this method allows evaluate the optimal number of purchased items under such constrains like the amount of money, available for a particular individual. Here, λ is the Lagrange multiplier, which plays a crucial role in the optimization process. It helps understand how the optimal value of the objective function changes as the constraint changes. In consumer choice problems, mentioned in this work, for instance, λ can be interpreted as the marginal utility of income, indicating how much additional utility is gained from an increase in budget (Chiang, 1967).

2. Methodology

Lagrange method, besides objective function, includes composition of Lagrange function by introducing a new variable λ which has to be multiplied on a conditional function, represented implicitly. For finding potential extrema points, the equation system is compiled, where first derivatives of variables x, y, λ become equal to nought. After various transformations are completed, all possible values of variables x, y, λ are found, so potential extrema points are formed. Then, first derivatives of the conditional function and second derivatives of Lagrange functions are found for the calculation of determinant 3*3. If it is more, than nought, extremum point is a maximum, if not, it becomes a minimum.

Furthermore, the Lagrange multiplier λ measures the sensitivity of the optimal value of the objective function to changes in the constraint. For instance, if the constraint increases, it indicates how the optimal solution is affected by a relaxation of the constraint. This sensitivity analysis is crucial for understanding the implications of changing constraints in real-world applications, such as resource allocation and production optimization (Chiang, 1967).

3. Development of a mathematical model, based on an example

3.1 Minimization of production expenses

Every day a plant manufactures 600 units of products x and y, using two different production technologies. Manufacturing costs of unit x, using first technology set as $20x^2$ function, but costs for unit y by second technology are $10y^2$. How many units of x and y should be manufactured to minimize expenses, utilizing each technology?

$$Z(x,y) = 20x^2 + 10y^2 \to min$$
$$\varphi(x,y) = x + y = 600$$
$$L(x,y,\lambda) = 20x^2 + 10y^2 + \lambda(x + y - 600)$$
$$\begin{cases} L'_X = 40x + 1 = 0 \\ L'_Y = 20y + 1 = 0 \\ L'_\lambda = x + y - 600 = 0 \end{cases}$$
$$\begin{cases} x = 400 \\ y = 200 \\ \lambda = 1/10 \end{cases}$$
$$M(400,200), \lambda = 1/10$$
$$L''_{xx} = 40$$
$$L''_{xy} = L''_{yx} = 0$$
$$L''_{yy} = 20$$
$$\varphi'_x = 1$$
$$\varphi'_y = 1$$
$$\triangleq \begin{vmatrix} 0 & \varphi'_x & \varphi'_y \\ \varphi'_x & L''_{xx} & L''_{xy} \\ \varphi'_y & L''_{yx} & L''_{yy} \end{vmatrix}$$
$$\triangleq \begin{vmatrix} 0 & 1 & 1 \\ 1 & 40 & 0 \\ 1 & 0 & 20 \end{vmatrix}$$
$$\triangleq -60 < 0 => minimal\ point$$
$$Z_{min}(200,400) = 20 \times 400^2 + 10 \times 200^2 = 3\ 600\ 000$$

3.2. Analysis of the obtained result

Due to the fact, that extremum point M (400,200) is a minimum of the objective function Z(x,y), there should be manufactured 400 units using the first technology and 200 units using the second one, to reach an optimal production plan and minimize costs (Bunu Ion et.al., 2012).

CONCLUSION

Nowadays, when competition on the market and resource is limited, production optimization becomes more important. Application of Lagrange method in Economics allows find optimal ways of manufacturing goods under limited resources, that contributes to decrease in expenses and increased productions efficiency. The results have shown that this method is useful when it comes to effective resource allocation. In the course of work, several mathematical models were developed and used to a particular manufacturing or purchasing process. Conditional extrema enabled determine optimum ratio of output and input costs.

Profitability of the enterprise directly depends on income-cost ratio and the adoption of such methods like mathematical modeling and data analysis enables minimization of total manufacturing expenditures through resource optimization. Such way of management has a positive impact on financial sustainability and competitiveness of the business on the market, allowing achieve increased production and efficiency in longer period of time.

Despite the advantages, mentioned above, application of Lagrange method requires profound knowledge in such areas like Applied Mathematics, Linear and Non-linear Programming and Economics. Furthermore, constant changes in market conditions should be considered to integrate mathematical model correctly.

Perspectives for development of this sphere are related to further integration of Lagrange method in modern technology, using artificial intelligence and machine learning, that give the possibility to manage resources more effectively.

REFERENCES:

1. Bunu, I., Vizitiu, V., Cracilo,v C. (2012). *Economic Mathematics.*
2. Chiang, A. C. (1967). *Fundamental Methods of Mathematical Economics.*
3. Joppen, R., von Enzberg, S., Kühn, A., Dumitrescu, R. (2019). *A practical Framework for the Optimization of Production Management Processes.* https://www.researchgate.net/publication/333468163_A_practical_Framework_for_the_Optimization_of_Production_Management_Processes
4. Kantorovich, L. (1960). *The Best Use of Economic Resources.*
5. Miniuc, S. (2002). *Mathematical Methods and Models in Economics.*

Coordonator ştiinţific: TACU Mariana,drd., asist. univ.,
Academia de Studii Economice din Moldova,
Republica Moldova, Chişinău, str. Bănulescu-Bodoni 61
e-mail: tacu_mariana@yahoo.com