Regional Economic Structure, Amenities and Disparities in an Extended Uzawa's Growth Model

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1. Introduction

An important phenomenon of modern economic activity is rapid agglomeration. Regional agglomeration has become increasingly more pronounced in different parts of the world. Nations, regions and cities grow at differential rates. A few metropolitan areas of the world are attracting more people. As explained by, for instance, by Bairoch (1993), rapid disparities between rich and poor regions are recent. Kuznets (1966) attributes this phenomenon to industrialization. In fact, the early development theories by Myrdal (1957) and Hirschman (1958) argue for dynamic interactions between industrial growth and the geographical concentration of industry: industrialization attracts resources to a given location and the resulting agglomeration stimulates growth. The contemporary literature on economic geography and economic development formalizes these dynamic processes in models based on different factors of nonlinear dynamics (e.g., Fujita et al, 1999, Forslid and Ottaviano, 2003, Zhang, 2008, 2009). Nevertheless, most of models in economic geography has not succeeded in including capital accumulation and infrastructures formation as endogenous processes of industrialization and agglomeration. This paper is concentrated on the study of interregional development with capital accumulation, taking account of factors such as environment and regional economic structure.

This study follows the neoclassical growth theory, emphasizing the role of wealth accumulation and amenity in regional growth and agglomeration. Capital accumulation is a key determinant of economic growth and development. Modern cities and metropolitan areas are formed with high concentration of buildings, infrastructures, and machines. Yet only a few formal models include capital accumulation in modelling dynamic economic geography with rational assumptions of profit and utility maximization. As pointed out by Zhang (2005), the traditional approaches to consumer behavior over time makes it analytically intractable to model spatial growth with rational consumer behavior. This study uses an alternative approach to consumer behavior in modeling a national economic growth. Our approach differs from the so-called new economic geography (e.g., Ottaviano *et al*., 2002; Forslid and Ottaviano, 2003; Pflüger, 2004; Charlot, 2006; Picard and Tabuchi, 2010). In almost all the dynamic models of the new economic geography, physical capital is completely neglected and regional amenities do not play a significant role in determining land rent and population mobility. Although this approach is claimed to have "enabled researchers to gain further insights into the space economy and its transition" (Tabuchi, 2014: 50). It is difficult to imagine

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any modern economy whose dynamics can be modelled neither with wealth nor with capital accumulation. To explain spatial economic agglomeration without taking account of spatial amenities tends to result in misleading results, as even understood from common sense. In the literature of the new economic geography, as Tabuchi (2014: 50) observes, "The scopes of most of the theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur." It is important to develop a model with any number of regions in order to address issues related to interregional growth and agglomeration. Many regions interact with each other in terms of trade and migration. This study introduces endogenous amenity in explaining regional agglomeration. Amenities have increasingly caused attention from spatial scientists (Glaeser *et al.*, 2001; Chen *et al.*, 2013). There is a large body of literature on amenities and spatial economics, for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown *et al.* (1997), life cycle studies by Clark and Hunter (1992), research on rural development by Deller *et al.* (2001). Zhang (1993) first introduced spatial amenity into utility in a general equilibrium framework. Zhang (1998) introduced spatial amenity into a formal regional growth model. Chen *et al.* (2013) developed a two-region model in which labor distribution, production externalities and natural resources are endogenous.

This paper is an extension of Zhang's two-region growth model (Zhang, 1996). This study generalizes the previous paper mainly by extending the two regions to any number of regions and simulates motion of the multi-regional economy rather than only examining the steady state. This study differs from Zhang (2008, 2009) in that the previous studies deal with the economies with only one sector in each region while in this paper each region has two sectors. This study differs from all the previous studies mentioned above by Zhang in that this paper succeeds in simulating the motion of the system and thus is able to conduct comparative dynamic analysis. This paper is organized as follows. Section 2 defines the multi-region model with capital accumulation and economic structure. Section 3 identifies the differential equations, which are be applied to simulate the model, plots the motion of the model, demonstrates the existence of a unique equilibrium point, and proves the stability of the equilibrium point. Section 4 carries out comparative dynamic analysis with regard to the total factor productivities of the two sectors, the propensity to save, the propensity to consume housing, and the relative amenity. Section 5 concludes the study. The main analytical results of Section 3 are proved in the Appendix.

2. Uzawa's two-sector growth model to a multi-regional economy

This paper is an extension of Zhang's dynamic two-region and two-sector trade model (Zhang, 1996). As in Zhang (1996), each region produces one goods and services. Most aspects of the goods sector in our model are similar to the neo-classical growth theory (Uzawa, 1961). There is only one (durable) good in the national economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; the available factors are fully utilized at every moment. Households undertake saving, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. Firms use all savings volunteered by households. We require saving and investment to be equal at any point in time.

The national economy consists of J regions, indexed by $j = 1, ..., J$. We assume perfect competition in all the markets both within each region and between the regions. Commodities are traded without any barriers. We neglect transport costs. We measure prices in terms of the commodity and the price of the commodity be unity. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th region. The interest rate is equalized throughout the national economy, i.e., $r(t) = r_j(t)$. The population N is homogenous. People are free to choose their residential location and people work and reside in the same region. Each region has fixed land L_j, which is homogenous within each region. The assumption of zero transportation cost of commodities implies price equality for the commodity among regions. As amenity and land are immobile, wage rates and land rent may vary between regions. We use subscripts, i, s, to denote the industrial and services sectors, respectively. Let $F_{jq}(t)$ stand for the output levels of q 's sector in region j at time t, $q = i$, s

Behavior of producers

We assume that there are two productive factors, capital, $K_{jq}(t)$, and labor, $N_{jq}(t)$, at each point in time t. The production functions are specified as:

$$F_{jq}(t) = A_{jq}\, K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \quad j = 1, ..., J, q = i, s.$$

We use $p_j(t)$ to stand for region j 's services price. Markets are competitive, thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rates are determined by markets. The production sector chooses the two variables, $K_{jq}(t)$ and $N_{jq}(t)$, to maximize its profit. The marginal conditions imply:

$$r(t)+\delta_{kj}=\frac{\alpha_{ji}F_{ji}(t)}{K_{ji}(t)}=\frac{\alpha_{js}\,p_j(t)F_{js}(t)}{K_{js}(t)},\;\; w_j(t)=\frac{\beta_{ji}F_{ji}(t)}{N_{ji}(t)}=\frac{\beta_{js}\,p_j(t)F_{js}(t)}{N_{js}(t)}, \tag{1}$$

where δ_{kj} are the depreciation rate of physical capital in region j.

Behavior of consumers

Each worker may get income from land ownership, wealth ownership and wages. In order to define incomes, it is necessary to determine land ownership structure. Land properties may be distributed in multiple ways under various institutions. To simplify the model, we accept the assumption of absent landownership, which means that the income of land rent is spent outside the economic system. A possible reasoning for this that the land is owned by the government, people can rent the land in competitive market, and the government uses the income for military or other public purposes. Consumers make decisions on choice of lot size, consumption levels of services and commodities as well as on how much to save. This study uses the approach to consumers' behavior proposed by Zhang in 1993 (see, Zhang, 2005). This approach makes it possible to solve many important (national) economic problems, such as growth problems with heterogeneous households, which are analytically intractable by the traditional approaches in economics.

Let $\bar{k}_j(t)$ stand for the per capita wealth in region j. The representative household of region j obtains income: $y_j(t)=\bar{r}(t)+r(t)\bar{k}_j(t)+w_j(t),$ (2)

from the interest payment and the wage payment. The total value of wealth that a consumer of region j can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is then equal to:

$$\hat{y}_j(t)=y_j(t)+\bar{k}_j(t). \tag{3}$$

The disposable income is used for saving and consumption. The value of wealth, $\bar{k}_j(t)$, is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we may consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time t by selling all of his wealth. Hence, at time t the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute between consuming and saving. At each point in time, a consumer distributes the total available budget among housing, $l_j(t)$, saving, $s_j(t)$, consumption of goods, $c_{ji}(t)$, and consumption of goods, $c_{js}(t)$. The budget constraint is given by:

$$R_j(t)l_j(t) + c_{ji}(t) + p_j(t)c_{js}(t) + s_j(t) = \hat{y}_j(t), \tag{4}$$

where $R_j(t)$ is region j's land rent. In our model, at each point in time, consumers have four variables to decide. A consumer decides how much to consume housing, goods and services, and how much to save. Equation (4) means that consumption and savings exhaust the consumers' disposable personal income.

We assume that utility level $U_j(t)$ that the consumers obtain is dependent on the consumption levels of lot size, commodity, services, and the saving. The utility level of the typical consumer in region j is:

$$U_j(t) = \theta_j(t)l_j^{\eta_0}(t)c_{ji}^{\xi_0} c_{js}^{\gamma_0}(t)s_j^{\lambda_0}(t), \ \ \eta_0, \gamma_0, \xi_0, \lambda_0 > 0, \tag{5}$$

in which η_0, ξ_0, γ_0, and λ_0 are a typical person's elasticity of utility with regard to lot size, commodity and savings in region j. We call η_0, ξ_0, γ_0, and λ_0 propensities to consume lot size, goods, and services, and to hold wealth (save), respectively. In (6), $\theta_j(t)$ is called region j 's amenity level. Amenities are affected by infrastructures, regional cultures and climates (e.g., Kanemoto, 1980; Diamond and Tolley, 1981; Blomquist, et al. 1988). In this study, we assume that amenity is affected by population. We specify θ_j as follows:

$$\theta_j(t) = \bar{\theta}_j N_j^d(t), \ \ j = 1, ..., J, \tag{6}$$

where $\bar{\theta}_j$ (> 0), d are parameters and $N_j(t)$ is region j 's population. We don't specify signs of d as population may have either positive or negative effects on regional attractiveness. As Chen *et al*. (2013: 269) observe: "The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households' migration decisions are internalized." We will examine effects of changes in amenity parameters on not only steady state but also transitory processes of the economic system.

Maximizing $U_j(t)$ subject to the budget constraints (5) yields

$$l_j(t)R_j(t) = \eta\, \hat{y}_j(t), \;\; c_{ji}(t) = \xi\, \hat{y}_j(t), \;\; p_j(t)c_{js}(t) = \gamma\, \hat{y}_j(t), \;\; s_j(t) = \lambda\, \hat{y}_j(t), \tag{7}$$

where $\eta \equiv \eta_0\, \rho, \;\; \xi \equiv \xi_0\, \rho, \;\; \gamma \equiv \gamma_0\, \rho, \;\; \lambda \equiv \lambda_0\, \rho, \;\; \rho \equiv \dfrac{1}{\eta_0 + \xi_0 + \gamma_0 + \lambda_0}.$

The saving behavior of the approach in this study is similar to these implied by the Keynesian consumption function and permanent income hypotheses, which are empirically more valid than the assumptions in the Solow model with a constant saving rate or the Ramsey model.

Wealth accumulation

According to the definitions of $s_j(t)$, the wealth accumulation of the representative person in region j is given by:

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{8}$$

Equalization of utility levels between regions

As households are freely mobile between the regions, the utility level of people should be equal, irrespective of in which region they live, i.e.

$$U_j(t) = U_q(t), \;\; j, q = 1, \ldots, J. \tag{9}$$

We don't take account of possible costs for migration. Changing houses or moving to another region will cost. Taking account of such changes in the model makes it difficult to simulate the model. Wage equalization between regions is often used as the equilibrium mechanism of population mobility over space. This study assumes that households obtain the same level of utility in different regions as the equilibrium mechanism of population distribution between regions.

Demand and supply balances

The total capital stocks $K(t)$ employed by the production sectors is equal to the total wealth owned by the households of all the regions. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t) N_j(t), \tag{10}$$

in which $K_j(t) \equiv K_{ji}(t) + K_{js}(t)$.

A region's supply of services is consumed by the region

$$c_{js}(t) N_j(t) = F_{js}(t), \;\; j = 1, 2. \tag{11}$$

Full employment of input factors

The assumption that labor force and land are fully employed is represented by

$$N_{ji}(t) + N_{js}(t) = N_j(t), \ \sum_{j=1}^{J} N_j(t) = N, \ l_j(t) N_j(t) = L_j, \ \ j = 1, \ldots, J. \quad (12)$$

We have thus built the model, which explains the endogenous capital accumulation and regional capital and labor distribution in the national economy in which all the markets are perfectly competitive and product, capital and labor are freely mobile.

3. Simulating the model

The dynamic system is complicated. For illustration, the rest of the study simulates the model. In the appendix, we show that the dynamics of the national economy can be expressed as $J+1$ dimensional differential equations. First, we introduce a variable $z_1(t)$

$$z_1(t) \equiv \frac{r(t) + \delta_{k1}}{w_1(t)}.$$

Lemma

The motion of the national economy is given by the following $J+1$ differential equations with $z_1(t)$ and $\left(\bar{k}_j(t)\right)$ as variables

$$\dot{\bar{k}}_j(t) = \Phi_j\left(z_1(t), \bar{k}_j(t)\right), \ \ j = 1, \ldots, J,$$
$$\dot{z}_1(t) = \Phi_0\left(z_1(t), \left(\bar{k}_j(t)\right)\right), \quad (13)$$

where Φ_j and Φ_0 are functions of $z_1(t)$ and $\left(\bar{k}_j(t)\right)$ defined in the appendix. For any given positive values of $z_1(t)$ and $\left(\bar{k}_j(t)\right)$ at any point in time, the other variables are uniquely determined by the following procedure: $\hat{y}_j(t)$ by (A8) $\rightarrow$ $r(t)$ by (A2) $\rightarrow$ $w_j(t)$ by (A4) $\rightarrow$ $p_j(t)$ by (A5) $\rightarrow$ $N_1(t)$ by (A10) $\rightarrow$ $N_j(t)$ by (A11) $\rightarrow$ $R_j(t)$ by (A12) $\rightarrow$ $l_j(t) = L_j / N_j(t)$ $\rightarrow N_{js}(t)$ by (A13) $\rightarrow$ $N_{ji}(t)$ by (A14) $\rightarrow$ $F_{jq}(t)$ by definition $\rightarrow$ $c_{ji}(t)$, $c_{ji}(t)$, and $s_j(t)$ by (7) $\rightarrow$ $K_{jq}(t)$ by (A1) $\rightarrow$ $K_j(t) = K_{ji}(t) + K_{js}(t)$ $\rightarrow$ $K(t)$ by (10) $\rightarrow$ $Y_j(t) = F_{ji}(t) + p_j(t) F_{js}(t)$ $\rightarrow$ $Y(t) = \sum_j Y_j(t)$.

Our dynamic equations are highly dimensional and nonlinear. The Lemma provides a computational procedure for following the motion of the economic system with any number of regions. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3 -region economy. We specify parameter values as follows:

$$N = 20, \ \lambda_0 = 0.75, \ \xi_0 = 0.1, \ \eta_0 = 0.1, \ \gamma_0 = 0.07, \ d = -0.05,$$

$$\begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.9 \end{pmatrix}, \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1 \\ 0.9 \end{pmatrix}, \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.31 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.32 \\ 0.32 \end{pmatrix}, \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 3.8 \\ 3.5 \\ 4 \end{pmatrix}, \begin{pmatrix} \delta_{k1} \\ \delta_{k2} \\ \delta_{k3} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.06 \end{pmatrix}. \tag{14}$$

Region 1 's levels of productivity of the two sectors are highest; region 2 's levels are the next; and region 3 's levels of productivity of the two sectors are lowest. We specify values of α_{jk} close to 0.3. With regard to the technological parameters, for illustration what are important in our interregional study are their relative values. The presumed productivity differences between the regions are not very large. It can be seen that the specified values of the land sizes, the preference parameters and the population will not affect our main concerns about interactions between the regions.

We specify the initial conditions as follows

$$z_1(0) = 0.09, \ \bar{k}_1(0) = 5.3, \ \bar{k}_2(0) = 4.6, \ \bar{k}_3(0) = 3.3.$$

The motion of the variables is plotted in Figure 1. The national output and national wealth rise over time till they arrive at the equilibrium level. The rate of interest falls in association with rising wealth. Region 1's total output and two sectors' output levels rises, region 2's total output and two sectors' output levels fall, and region 3's total output and two sectors' output levels are slightly changing. People migrate from region 2 to region 1. Region 1's amenity slightly falls, and region 2's amenity is improved, and region 3's amenity is slightly affected. The service prices and wage rates are slightly changing over time. Although the national economy and region 1's economy are improving, region 2' economy falls, and region 3's economy is almost stationary.

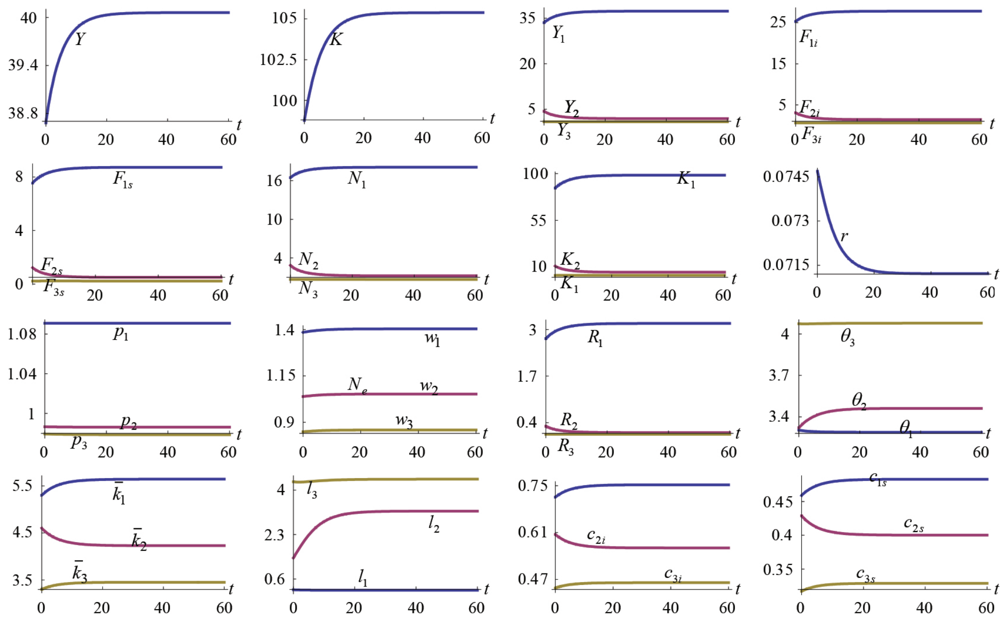


Figure 1. The Motion of the Economic System

It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (22)

$$Y = 41.6, \quad K = 109.6, \quad r = 0.068,$$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 37.8 \\ 1.93 \\ 0.84 \end{pmatrix}, \quad \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 18.08 \\ 1.25 \\ 0.67 \end{pmatrix}, \quad \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 28.28 \\ 1.44 \\ 0.62 \end{pmatrix}, \quad \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 8.73 \\ 0.5 \\ 0.22 \end{pmatrix}, \quad \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 13.53 \\ 0.93 \\ 0.5 \end{pmatrix},$$

$$\begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 4.56 \\ 0.31 \\ 0.17 \end{pmatrix}, \quad \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 76.68 \\ 3.78 \\ 1.45 \end{pmatrix}, \quad \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 25.82 \\ 1.33 \\ 0.53 \end{pmatrix}, \quad \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.09 \\ 0.98 \\ 0.97 \end{pmatrix}, \quad \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1.42 \\ 1.06 \\ 0.87 \end{pmatrix},$$

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} 3.17 \\ 0.13 \\ 0.07 \end{pmatrix}, \quad \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 3.29 \\ 3.46 \\ 4.08 \end{pmatrix}, \quad \begin{pmatrix} \bar{k}_1 \\ \bar{k}_1 \\ \bar{k}_1 \end{pmatrix} = \begin{pmatrix} 5.64 \\ 4.23 \\ 3.44 \end{pmatrix}, \quad \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 0.17 \\ 3.2 \\ 4.48 \end{pmatrix}, \quad \begin{pmatrix} c_{1i} \\ c_{2i} \\ c_{3i} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.56 \\ 0.6 \end{pmatrix}, \quad \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 0.48 \\ 0.4 \\ 0.33 \end{pmatrix}. \quad (22)$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.1911, \quad -0.1908, \quad -0.1908\}.$$

The three eigenvalues are real and negative. The unique equilibrium is locally stable. This guarantees the validity of exercising comparative dynamic analysis.

4. Comparative dynamic analysis

We simulated the motion of the national economy under (21). We now study how the economic system reacts to changes, for instance, in the preference. As the lemma gives a computational procedure to calibrate the motion of all the variables, it is straightforward to conduct comparative dynamic analysis. In the rest of this study we use $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter value.

4.1. The total factor productivity of region 2's industrial sector rises

We first study the effects of a technological improvement in region 2's industrial sector. The technological progress is specified as follows: $A_{2i}: 1 \Rightarrow 1.05$. The simulation results are plotted in Figure 2. It should be noted that the value of A_{2i} after the change is still lower than the value of A_{1i}. This implies that if all the other conditions being the same, then migration from region 1 to region 2 may reduce the national output. The national output is enhanced initially but lowered in the long term. Different from the national income national wealth is increased. To explain this difference we note that as region 2's technological progress attracts more people to the region from the other two regions. As region 1's total factor productivity is higher than region 2's and region 2's is higher than region 3' and more people migrate from region 1 than region 3, we see the possibility that the net impact on the national income due to redistribution of labor force is negative. Region 2's lot size falls and land rent rises; the other two regions' lot sizes are increased and land rents are reduced. Region 2's service prices, wage rate, consumption of the industrial goods and wealth per capita are increased, while these variables are slightly affected in the other two regions. The service consumption level in the region 2 falls initially and then rises. This happens as the price rises rapidly but the rises in the wage and wealth take a longer time before the net impact on service consumption becomes positive. Another insight we obtain from this analysis is about dynamics of wage disparities over time between regions. Wage disparities are caused by many factors, such as spatial differences in education opportunities, knowledge diffusion, skill composition of the workforce, local interactions, discrimination, as well as non-human endowments (for instance, Glaeser and Maré, 2001; Duranton and Monastiriotis, 2002; Rey and Janikas, 2005). From our simulation result, we see that the wage disparity is strongly affected by change in technology. This also hints that if technological differences between regions are not large, wage rates may tend to converge if the other factors weakly affect the differences.

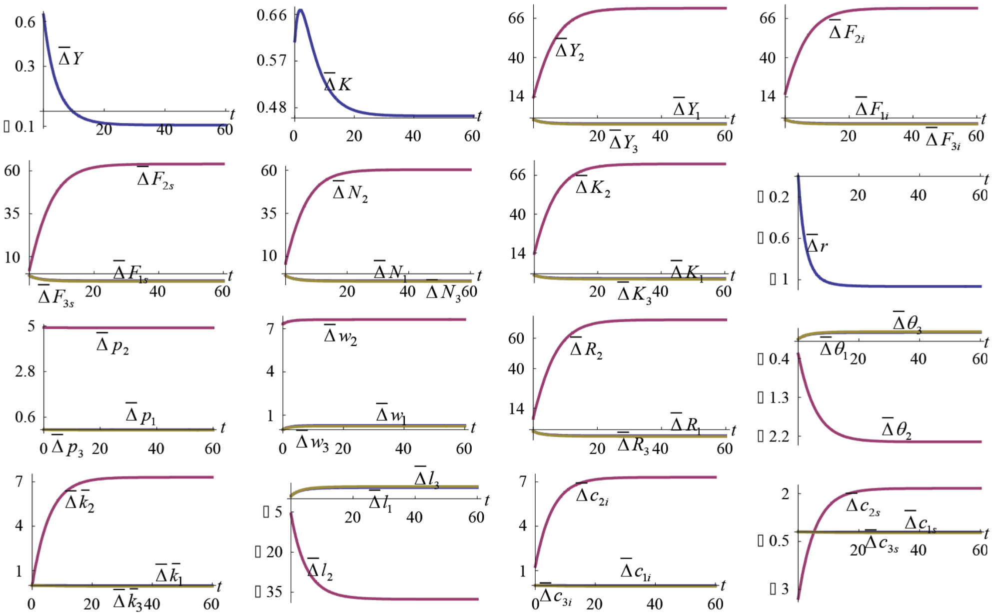


Figure 2. The Total Factor Productivity of Region 2's Industrial Sector Rises

4.2. The total factor productivity of region 2's service sector rises

We now examine the effects of the following technological improvement in region 2's service sector: $A_{2s}: 1 \Rightarrow 1.05$. The simulation results are plotted in Figure 3. The national output and wealth are slightly reduced by the technological improvement. This happens as people migrate from the advanced region to a less technologically advanced region, leading to the reduction of the national income and wealth. Region 2's technological progress attracts more people to the region from the other two regions. Region 2's lot size falls and land rent rises; the other two regions' lot sizes are increased and land rents are reduced. No region's wage rate is increased. This occurs partly because the total capital is reduced. The reduction in the total capital is associated with rising in the rate of interest. Region 2 attracts more capital while the other two regions have less capital employed. The output levels of Region 2's two sectors are enhanced. The output levels of the industrial sectors in the other two regions are lowered. The regional income of region 2 rises and the other two regions' regional income levels fall. Region 2's service price falls and consumption of services rises, while these variables are slightly affected in the other two regions. The wealth levels are slightly affected. It is interesting to compare Figures 2 and 3. The effects on wealth per households, wage rates, prices of services, and national income and wealth are different in the two cases. The results for Figure 2 are held for the sector whose product is freely mobile, while the results for Figure 3 are for the sector whose production has to be consumed in the region. There are different studies on regional economic growth with endogenous knowledge (Florida et al., 2008; Fleisher, et al., 2010). Although our study does not include endogenous technological change, the literature of regional economic growth and knowledge should enable us to generalize our modeling.

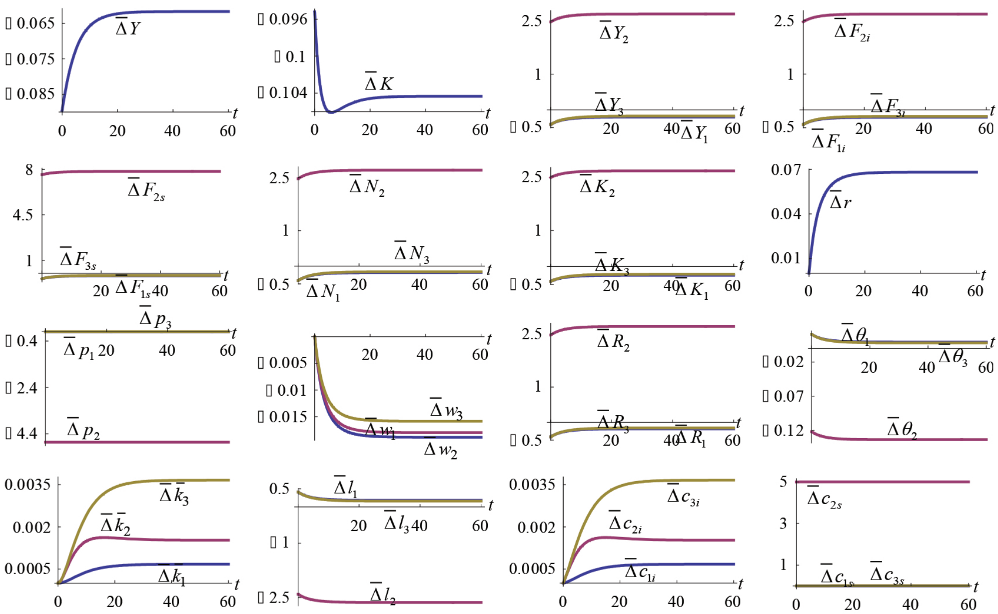


Figure 3. The Total Factor Productivity of Region 2's Service Sector Rise

4.3. A rise in the propensity to save and spatial agglomeration

In economic growth theory effects of saving propensity changes are different in different theories. In Keynesian economic theory, savings tend to reduced national income, while neoclassical growth theory tends to suggest the opposite effect. As only a few growth models with space take account of endogenous savings, regional growth theory has not much to say on how a change in the propensity to save can affect spatial agglomeration and regional economic growth. We now allow the propensity to save to be changed as follows: $.75 \Rightarrow 0.77$. The simulation results are plotted in Figure 4. The national output and wealth are increased. The rate of interest is lowered. The change in the propensity to save has a strong impact on regional disparity and population distribution. As the economy has more capital, region 1 attracts more people from the other two regions. This results in enlarged differences between region 1 and the other two regions. Region 1's regional income is increased, while the other two regions" regional income levels are reduced. Region 1 employs more capital while the other two regions us less capital. The two sectors' output levels in region 1 are reduced while the two sectors' output levels in the other two regions are enhanced. The wage rates and wealth levels per household in all the regions are increased. Region 1's amenity is deteriorated, while the amenity levels in the other regions are improved. The consumption levels of both industrial goods and services in all the regions fall initially and then increased. This happens as the households save more out of disposable income much and income and wealth are not yet increased much initially, the consumption levels are reduced. As the households have more wealth and higher wage incomes, the consumption level are increased in the long term.

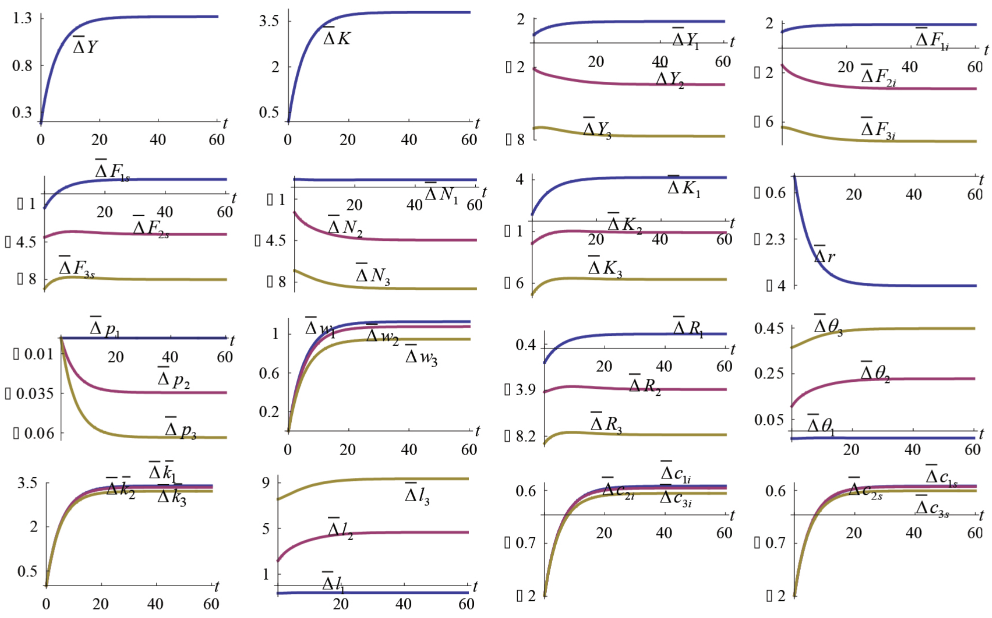
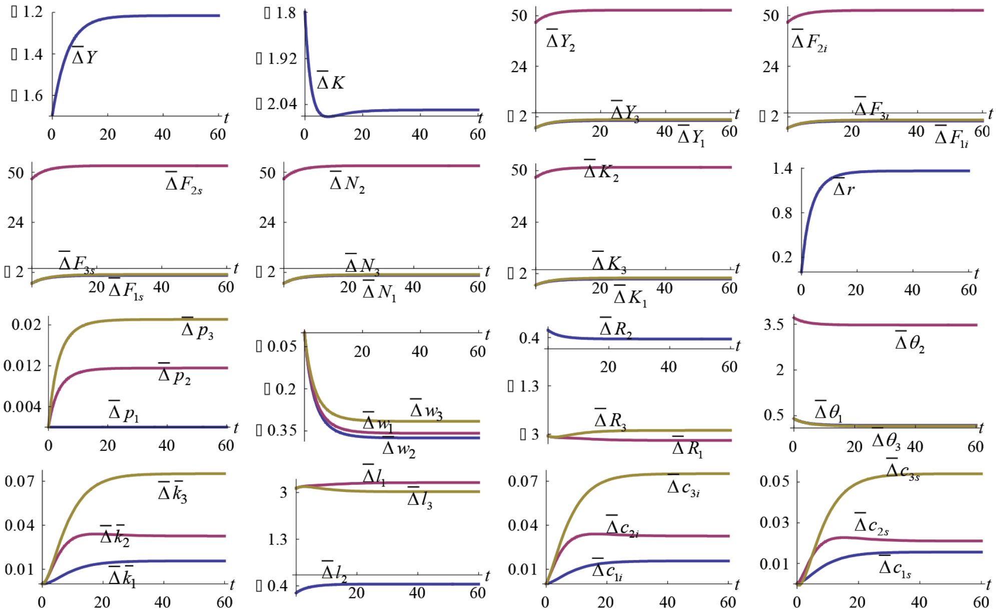


Figure 4. A Rise in the Propensity to Save and Spatial Agglomeration

4.4. Regional amenity and spatial agglomeration

We now analyze the effects of the following rise in region 2's amenity parameter: $\bar{\theta}_2 : 3.5 \Rightarrow 3.7$. The simulation results are plotted in Figure 5. The national output and wealth fall as people prefer more strongly to living in the less advanced region. The rate of interest rises and the wage rates in all the regions fall. Region 2 attracts more people from the other two regions and its total income, the output of the two sectors, and capital employed by the region are all increased, while the corresponding variables in the other two regions are all reduced. Region 2's amenity is improved while the other two regions' amenity levels are enhanced but slightly. The consumption levels and prices are affected but all very slightly. It should be noted that our approach on regional housing markets can be related to hedonic price modelling (e.g., Rosen, 1974; Helbich et al., 2014). The approach is based in Lancaster's idea that it is a good's characteristics that creates utility. When we apply this idea to housing market which are tied with environment and land, it implies that environment should have effects on housing prices (Dubin, 1992; Malpezzi, 2003; Ahlfeldt, 2011). Our model shows how the rent levels are closely related to different regional characteristics.



4.5. The propensity to consume housing and spatial agglomeration

We now study the effects of the following rise in the population's propensity to consume housing: $\bar{\theta}_2 : 3.5 \Rightarrow 3.7$. The simulation results are plotted in Figure 6. The national output and wealth fall as people devote more out of their disposable income to housing. The rate of interest rises and the wage rates in all the regions fall. Regions 2 and 3 attract more people from region 1. The land rents in all the regions are increased. Even when region 1 has less residents, its land rent rises as well. Region 1's total income, the output of the two sectors, and capital employed by the region are all reduced, while the corresponding variables in the other two regions are all enhanced. Region 1's amenity is improved while the other two regions' amenity levels are deteriorated. The consumption levels and wealth levels per household are reduced in all the regions.

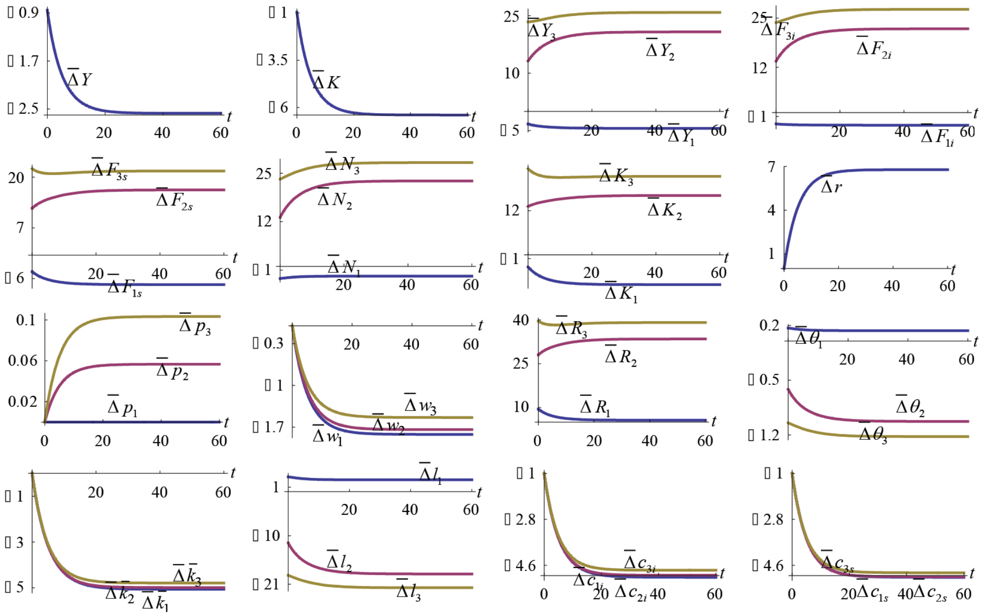


Figure 6. A Rise in the Propensity to Consume Housing

5. Conclusions

This paper extended Uzawa's two-sector growth model to any number of regions. In the model capital accumulation and endogenous amenity are endogenous. The economy is built under assumptions of profit maximization, utility maximization, and perfect competition. We used the utility function proposed by Zhang (2005) to determine saving and consumption. The dynamics of J region national economy is controlled by $J+1$ differential equations. We simulated the model with a 3-region model and demonstrated the existence of a unique equilibrium point. Our comparative analysis provides some important insights. As the model is structurally general, it is possible to deal with various national as well as regional growth and environment issues. It is straightforward to analyze behavior of the model with other forms of production or utility functions. Households should be heterogeneous. Also issues related to tax competition between regions have caused great attention in economic geography. We can extend the dynamic equilibrium framework to examine these issues.

Appendix: Proving the Lemma

We now show a procedure to determine the dynamics of the system in two differential equations with general production functions. First, from equations (2) we obtain

$$z_j \equiv \frac{r + \delta_k}{w_j} = \frac{a_j N_{ji}}{K_{ji}} = \frac{b_j N_{js}}{K_{js}}, \tag{A1}$$

where

$$a_j \equiv \frac{\alpha_{ji}}{\beta_{ji}}, \;\; b_j \equiv \frac{\alpha_{js}}{\beta_{js}}.$$

Insert $z_j / a_j \equiv N_{ji} / K_{ji}$ in $r + \delta_{kj} = \alpha_{ji} F_{ji} / K_{ji}$ from (1)

$$r(z_j) = \frac{\alpha_{ji} A_{ji}}{a_j^{\beta_{ji}}} z_j^{\beta_{ji}} - \delta_{kj}, \;\; j = 1, ..., J. \tag{A2}$$

From (A2) we get

$$z_j(z_1) = a_j \left(\frac{r + \delta_{kj}}{\alpha_{ji} A_{ji}} \right)^{1/\beta_{ji}}, \;\; j = 2, ..., J. \tag{A3}$$

From (A1) and (A2), we have

$$w_j(z_1) = \frac{r + \delta_k}{z_j}. \tag{A4}$$

From $z_j = b_j N_{js} / K_{js}$ and (1), we have

$$p_j(z_1) = \frac{b_j^{\beta_{js}} (r + \delta_k)}{\alpha_{js} A_{js} z_j^{\beta_{js}}}. \tag{A5}$$

From (11) and $p_j c_{js} = \gamma \hat{y}_j$ we have

$$\gamma \hat{y}_j N_j = p_j F_{js}. \tag{A6}$$

Insert (1) in (A6)

$$\gamma \hat{y}_j N_j = \frac{w_j N_{js}}{\beta_{js}}. \tag{A7}$$

By (3) we have

$$\hat{y}_j(z_1, \bar{k}_j) = (1 + r)\bar{k}_j + w_j. \tag{A8}$$

Substitute $l_j = L_j / N_j$, $\theta_j = \bar{\theta}_j N_j^d$, and (7) into (6)

$$U_j = \frac{\bar{\theta}_j N_j^{d-\eta_0} L_j^{\eta_0}}{p_j^{\gamma_0}} \xi^{\xi_0} \gamma^{\gamma_0} \lambda^{\lambda_0} \hat{y}_j^{\xi_0+\gamma_0+\lambda_0}, \tag{A9}$$

Apply $U_j = U_q$ to (A9)

$$N_j = \Lambda_j N_1, \tag{A10}$$

where

$$\Lambda_j(z_1, (\bar{k}_q)) \equiv \left(\frac{\bar{\theta}_1 L_1^{\eta_0} p_j^{\gamma_0}}{\bar{\theta}_j L_j^{\eta_0} p_1^{\gamma_0}}\right)^{\frac{1}{(d-\eta_0)}} \left(\frac{\hat{y}_1}{\hat{y}_j}\right)^{\frac{\xi_0+\gamma_0+\lambda_0}{d-\eta_0}}.$$

Insert (A10) in (13)

$$N_1(z_1, (\bar{k}_q)) = \frac{N}{\sum_{j=1}^{J} \Lambda_j}, \quad \Lambda_j = 1. \tag{A11}$$

With (A10) and (A11) we determine the population distribution as functions of z_1 and $(\bar{k}_q)$. By $l_j R_j = \eta \hat{y}_j$ and $l_j N_j = L_j$, we have

$$R_j\left(z_1, \left(\bar{k}_q\right)\right) = \frac{\eta \, \hat{y}_j \, N_j}{L_j}. \tag{A12}$$

Insert (A8) in (A7)

$$N_{js}\left(z_1, \left(\bar{k}_q\right)\right) = \left(\frac{(1+r)\bar{k}_j}{w_j} + 1\right)\gamma \, \beta_{js} \, N_j . \tag{A13}$$

From $N_{ji} + N_{js} = N_j$ and (A10), we have

$$N_{ji}\left(z_1, \left(\bar{k}_q\right)\right) = N_j - N_{js}, \quad j = 1, \ldots, J. \tag{A14}$$

From equation (11), we have

$$\sum_{j=1}^{J}\left(K_{ji} + K_{js}\right) = \sum_{j=1}^{J} \bar{k}_j N_j . \tag{A15}$$

Insert (A1) in (A15)

$$\sum_{j=1}^{J}\left(\frac{a_j \, N_{ji} + b_j \, N_{js}}{z_j}\right) = \sum_{j=1}^{J} \bar{k}_j N_j . \tag{A16}$$

Insert (A13) and (A14) in (A16)

$$\sum_{j=1}^{J}\left(\frac{\left(b_j - a_j\right)}{z_j}\left(\frac{(1+r)\bar{k}_j}{w_j} + 1\right)\gamma \, \beta_{js} \, N_j\right) = \sum_{j=1}^{J}\left(\bar{k}_j - \frac{a_j}{z_j}\right)N_j . \tag{A17}$$

Insert (A10) in (A17)

$$\Phi\left(z_1, \left(\bar{k}_j\right)\right) \equiv \sum_{j=1}^{J}\left(\left(\frac{(1+r)\bar{k}_j}{w_j} + 1\right)\frac{B_j}{z_j} - \bar{k}_j + \frac{a_j}{z_j}\right)\Lambda_j = 0, \tag{A18}$$

where $B_j \equiv \left(b_j - a_j\right)\gamma \, \beta_{js}$.

Substitute $s_j = \lambda \hat{y}_j$ and $r\bar{k}_j + w_j$ into equations (9)

$$\dot{\bar{k}}_j = \Phi_j\left(z_1, \bar{k}_j\right) \equiv (1+r)\lambda \bar{k}_j + \lambda w_j - \bar{k}_j . \tag{A19}$$

Taking derivatives of equation (A18) with respect to t yields

$$\dot{z}_1 = -\left(\sum_{j=1}^{J} \frac{\partial \Phi}{\partial \bar{k}_j} \dot{\bar{k}}_j\right)\left(\frac{\partial \Phi}{\partial z_1}\right)^{-1} . \tag{A20}$$

Insert (A19) in (A20)

$$\dot{z}_1 = \Phi_0\left(z_1, \left(\bar{k}_j\right)\right) \equiv -\left(\sum_{j=1}^{J} \Phi_j \frac{\partial \Phi}{\partial \bar{k}_j}\right)\left(\frac{\partial \Phi}{\partial z_1}\right)^{-1} . \tag{A21}$$

Following the procedure in the Lemma we describe the dynamics of the whole system.

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